International Journal of Physics
and Research (IJPR)
ISSN (P): 2250-0030; ISSN (E): 2319-4499
Vol. 9, Issue 1, Jun 2019, 9-20

GARDNER SOLITONS IN ELECTRON-POSITRON-ION PLAMSA FEATURING CAIRNS-TSALLIS ELECTRONS

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ABSTRACT

In the present study, the effects of non-thermal and nonextensive distribution of electrons on the soliton propagation in plasma system containing Boltzmann positrons have been studied. Using Reductive Perturbation method, Korteweg-de Vries (K-dV), modified K-dV (mK-dV) and Gardner equations are derived for electron-positron-ion (e-p-i) plasma system. The soliton solution of the Gardner equation is discussed in detail. Results have been interpreted in the form of graphs. It is found that for a given set of parameters, there exists a critical value of q (i.e, qc) below which only rarefactive K-dV solitons exist and above it compressive K-dV solitons exist. However, both positive and negative mK-dV and Gardner potential structures exist at the critical value of nonextensivity. The present investigation may help us to understand the electrostatic perturbations in laboratory and space plasmas.

KEYWORDS: Nonlinear Waves, Nonthermal Distribution, Tsallis Distribution, Solitons, Reductive Perturbation Method & Electron-Positron-Ion (e-p-i) Plasma

Received: Jan 10, 2019; **Accepted:** Jan 30, 2019; **Published:** Feb 14, 2019; **Paper Id.:** IJPRJUN20192

Original Article

1. INTRODUCTION

There are manifests of positron component (that have an equal mass but different charge to electrons) in an ordinary electron-ion plasma, forming an electron-positron-ion (e-p-i) plasma. The existence of positrons can be found both in laboratory and natural environments, such as the neutron stars (Michel, 1991), the active galactic nuclei (Miller and Witta, 1987), and at the center of the Milky Way galaxy (Barns, 1983). The positrons can also be produced in laser plasma experiments when ultra-intense laser pulse interacts with the matter. That was why the wave propagation in e-p-i plasma has become an interesting focus point to the plasma physics researchers over the last few decades. Already a large number of authors have studied the properties of linear and nonlinear waves in e-p-i plasmas by considering different plasma models. However, most of the investigations are limited to the Korteweg de Vries (K-dV) equation, either in planar or in nonplanar geometry in e-p-i plasmas. Parametric regime may create a situation for which the nonlinear term, noted A, of the K-dV equation is vanishingly small, i.e. $A \sim 0$. In this case, the nonlinear term may give rise to very large amplitude structures, breaking down the validity of the reductive perturbation method (Washimi and Taniuti, 1966).To get rid of this circumstance, i.e., to study finite amplitude ion-acoustic solitary waves (IASWs) beyond the K-dV limit, one may derive another type of nonlinear dynamical equation valid for A □0. Recently, some focus has been given by many researchers on Gardner (higher order K-dV equation or mixed K-dV equation) or modified Gardner equation (Wazwaz, 2007; Vassilev et al., 2011; Hossain et al., 2009; Mamun and Islam, 2011). Mannan and Mamun (2011) have studied the nonlinear propagation of Gardner solitons (GSs) in a nonplanar four-component dusty plasma. They derived the modified Gardner (MG) equation and solved it numerically. Shan and Akhtar(2013)have derived the K-dV equation for ion

acoustic soliton with negative ions in the presence of nonextensive electrons. Ferdousi et al. (2014)have studied the ion acoustic K-dV, mK-dV, and Gardner solitons in unmagnetized e-p-i plasma with nonextensive electrons and positrons. A theoretical investigation for ion-acoustic Gardner solitons and double layers in a multi-ion plasma system was presented by El-Taibany et al. (2018).

Non-Maxwellian particle distribution functions are commonly observed both in space and laboratory plasmas. For example, the observations made by the Viking spacecraft (Bostrom, 1992) and Freja satellite (Dovner et al., 1994) have found electrostatic solitary structures in the Earth's magnetosphere with density depressions. Motivated by these events, Cairns et al.(1995) showed that the presence of nonthermal electrons changes the nature of the ion sound solitary structures and allows the existence of both positive and negative ion-acoustic (IA) solitary structures, like those observed by Freja and Viking. Various observations of fast ions and electrons in interplanetary space environments clearly indicate that these particles have velocity distribution functions that deviate from the Maxwellian behaviour(Asbridge et al.,1968; Divine and Garret, 1983; Krimigis et al., 1983; Lundlin et al., 1989; Futaana et al., 2003; Shan et al., 2014; Akhtar et al., 2014). Therefore, the Maxwellian distribution may be inadequate to describe the long range interactions in plasmas in the presence of nonequilibrium stationary states because the Maxwellian distribution is valid only for the macroscopic ergodic equilibrium state. Furthermore, observations show that astrophysical and space plasmas have a particle distribution function which are quasi-Maxwellian up to the mean thermal velocities, and possess non-Maxwellian tails at high velocities or energies. The Tsallis distribution in nonextensive statistical mechanics as a prominent example of a non-Maxwellian distribution function has been applied to study ion-acoustic waves (IAWs) in such plasmas, including the solar interior and the interplanetary medium (Liu and Du, 2009). Moreover, more evidence has shown that the Tsallis nonextensive statistics (Tsallis, 1988) may be very important for systems endowed with long-range interactions as usually happens in astrophysics and plasma physics (Du, 2004; Liyan and Du, 2008;Liu et al., 2009; Tribeche and Djebarni, 2010; Amour and Tribeche, 2010; Gougam and Tribeche, 2011; Bains et al., 2011; Tribeche and Merriche, 2011; Moghanjoughi, 2011). Numerous observations clearly indicated the presence of energetic particles as ubiquitous in a variety of astrophysical plasma environments, and measurements of their distribution functions revealed them to be highly nonthermality(Goldman et al., 1999).

Tribeche et al. (2012) compared the nonthermal and nonextensive distributions and found that the two distributions may act concurrently on the nature of ion-acoustic waves (IAWs). Hence they proposed a hybrid Cairns-Tsallis distribution function, which offers enhanced parametric flexibility in modeling nonthermal plasmas, as in principle such a two-parameter representation of the distribution function could be useful in fitting a wider range of observed plasmas. Motevalli et al. (2018) had studied ion- and positron-acoustic solitons in magnetized dusty plasma with q-nonextensive electron and positron velocity distribution. Moreover, Williams et al. (2013) employed the Sagdeev pseudo potential method to investigate the large amplitude IAW dynamics in an electron–ion plasma with the Cairns-Tsallis ($0.6< q\leq 1$ and $0\leq \alpha< 0.25$) electron distribution function. They determinedthe validity range of the model and defined the existence ranges for the solitons supported by this distribution.Our aim here is to trace the influence of electrons and positrons nonextensivity and nonthermality, especially positron to electron number density (p) and free electron to positron temperature ratio (δ) on the propagation of IASWs in unmagnetized e-p-i plasma. The paper is organized as follows: Derivation of K-dV and mK-dV equations is provided in section 2 along with their numerical discussion. Section 3 deals with the derivation and solution of Gardner equation for the given plasma system. The conclusion of this study is provided in section 4.

2. K-dV AND MODIFIED K-dV EQUATIONS

The nonlinear propagation of finite amplitude ion acoustic waves in a three component unmagnetized, collisionless plasma consisting of Boltzmann positrons, cold ions and nonthermal-nonextensive electrons have been considered here. The cold ion fluid and positrons are described by fluid equations, while the electrons are assumed to satisfy hybrid Cairns-Tsallis velocity distribution. Thus, in equilibrium, the charge neutrality condition is $n_{eo} = n_o + n_{po}$ where n_{eo}, n_{po} and n_o are the unperturbed number densities of the electron, positron and ion respectively. The dynamics of IA waves in such e-p-i plasma can be described by the following set of normalized fluid equations:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu n_e + (1-\mu)n_p - n = \rho \tag{3}$$

Where n, n_p and n_e are the normalized ion, positron and electron number densities respectively. Also, number densities of ions n, positron n_p and electron n_e are normalized by unperturbed electron number density n_{eo}. In the above equations, u is the ion fluid velocity which is normalized by ion acoustic speed, $C_s= (K_BT_{ef}/m)^{1/2}$; ϕ is the electrostatic potential normalized by thermal potential K_BT_{ef}/e. The time (t) and space coordinate (x) have been normalized with respect to inverse of ion-plasma frequency in the mixture$\omega_{pi}^{-1}=\lambda_D/C_s$ and Debye length $\lambda_D = (\varepsilon_0 K_B T_{ef}/n_{e0}e^2)^{1/2}$ respectively. Here, K_B is the Boltzmann constant, T_{ef} is the constant free electron temperature and e is the electronic charge. The normalized electron number density with nonthermal-nonextensive distribution is given by:

$$n_e = \left(1 + F_1\phi + F_2\phi^2\right)\times\left(1+(q-1)\phi\right)^{\frac{(q+1)}{2(q-1)}} \tag{4}$$

Where q is the nonextensivity parameter. Here, $F_1 = -16q\alpha/(3-14q+15q^2+12\alpha)$, $F_2 = 16(2q-1)q\alpha/(3-14q+15q^2+12\alpha)$, the parameter α stands for the number of nonthermal electrons in the distribution, and *q*(> -1) measures the strength of nonextensivity. If α=0, the distribution (4) reduces to the q-nonextensive distribution $n_e = (1 + (q-1)\phi)^{(q+1)/2(q-1)}$. If $q\to 1$, equation (4) recovers to the well-known Cairns distribution $n_e = [1 -4\alpha\phi/(1+3\alpha) +4\alpha\phi^2 /(1+3\alpha)]\exp(\phi)$. The normalized positron number density is given by $n_p = e^{-\delta\phi}$ where the parameters are given by $\mu = 1/(1-p)$, $p = n_{po}/n_{eo}$ and $\delta = T_{ef}/T_p$.

To study the ion acoustic Gardner solitons in e-p-i plasma, we use the reductive perturbationtechnique and first introduce the stretched coordinates (Washimi and Taniuti, 1966) as $\xi=\varepsilon^{1/2}(x-v_pt)$ and $\tau=\varepsilon^{3/2}t$. Here ε is a small parameter (0 <ε<1) measuring the weakness of the dispersion, and v_p is the phase velocity of the perturbation mode. Next, we expand all the dependent variables about their equilibrium values in powers of ε as follows:

$$X = X_0 + \sum_r \varepsilon^r X^{(r)} \tag{5}$$

Here X = n, u, ϕ, ρ and X_0 = 0,1,1,1. Using stretched coordinated and the expansion (5) into Equations (1) – (4) and equating terms with the same powers of ε, we obtain a set of equations for each order in ε.The set of Equations

(1) – (3) at the lowest order, i.e., o (ε), give $u^{(1)} = \phi^{(1)}/v_p$, $n^{(1)} = \phi^{(1)}/v_p^2$ and $v_p = \sqrt{(1-p)/(c_1 + p\delta)}$. Here, $c_1 = F_1 + a_1$ and $a_1 = (q + 1)/2$ respectively. Equation of v_p gives the dispersion relation for ion-acoustic solitary waves. In order to get a combined effect of nonextensivity and non-thermality, the three dimensional view of variation of phase velocity (v_p) as a function of nonextensivity (q) and non-thermality (α) is shown in figure 1(a), with other parameters as p = 0.1 and δ = 1. It is observed that phase velocity of solitary waves increases with nonthermality and decreases with nonextensivity. It is found that the phase velocity decreases with nonextensivity (q) which is also clear from the mathematical relation of phase velocity where v_p is inversely proportional to nonextensivity (q).Similar kind of behavior has been observed by Sahoo et al. (2015) and Akhtar et al. (2013) in their respective researches.

By equating the next higher order coefficients of ε, one can obtain the following K-dV equation:

$$\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B\frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{6}$$

We have replaced $\phi^{(1)}$ with ϕ for the sake of simplicity. The nonlinear (A) and dispersion (B) coefficients given as

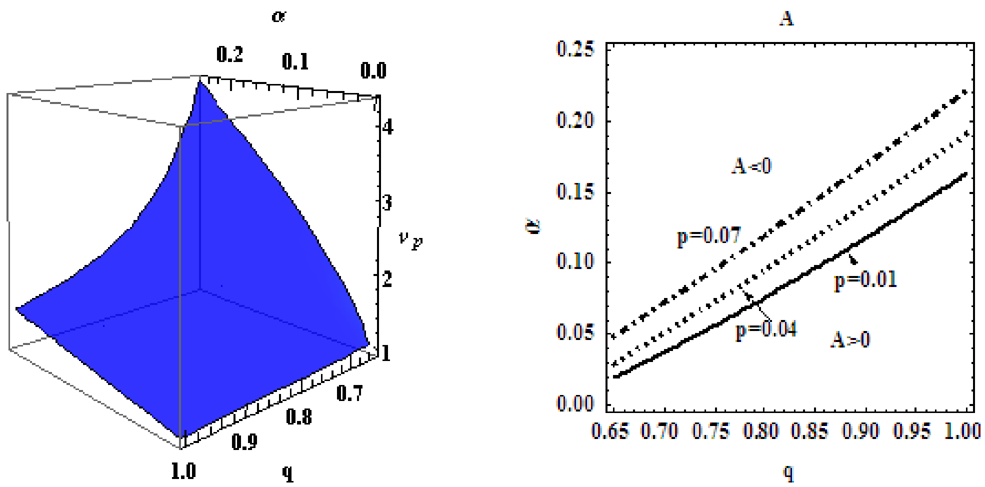

Figure 1: (a) 3D Profile of Wave Phase Velocity (vp) as a Function of Nonextensivity (q) and Nonthermality (α) with p = 0.03 and δ = 1
(b) Contour Plot of Nonlinear Coefficient (A) in (α-q) Space with Different p =0.01 (Solid Line), p = 0.04 (Dotted Line), p = 0.07 (Dashed Line) with δ = 1

$$A = B\left[\frac{3}{v_P^4} - 2c_2\mu - (1-\mu)\delta^2\right] \text{ and } B = \frac{v_p^3}{2}$$

Here, $c_2 = F_2 + F_1a_1 + a_2$ and $a_2 = (q+1)(3-q)/8$. Figure 1(b), represents the contour plot of nonlinear coefficient of K-dV equation (6) i.e. A in (α-q) space with their different values of positron to electron number density p = 0.01 (solid line), p = 0.04 (dotted line) and p = 0.07 (dashed line). Region of dip (hump) shaped structure increases with decrease (increase) in positron density. It may be further mentioned that for p > 0.1, the three regions of existence of solitary waves appear corresponding to two critical values of nonextensivity. The steady state localized solution of K-dV equation (6) can be shown to be

$$\phi_1 = \phi_{1m} \sec h^2\left(\frac{\eta}{W_1}\right) \tag{7}$$

Where the amplitude ϕ_{1m} and width W_1 are given by $\phi_{1m} = 3U_0/A$ and $W_1 = (4B/U_0)^{1/2}$ respectively. We also note that for typical plasma parameters p = 0.01, δ = 1, α = 0.1, A > 0 for q > 0.859051 and A < 0 for q < 0.859051. Here, A> 0 corresponds to hump shaped structure and region A < 0 corresponds to dip shaped ones. At critical value of q i.e., q_c potential structures do not exist.

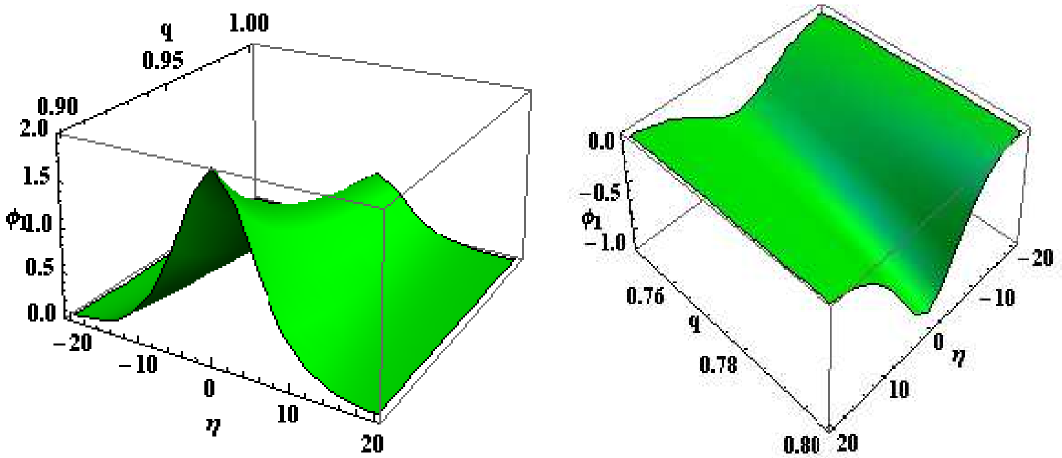

Figure 2: For p = 0.01, δ = 1,α = 0.1and U_0 = 0.10, 3D-Variation of the (a) Positive Potential Structure (ϕ_1) of K-dV Soliton with the Nonextensive Parameter, q (q >qc) (b) Negative Potential Structure (ϕ_1) of KdV Soliton with the Nonextensive Parameter, q (q <qc)

From the 3D profiles given in figures 2(a) and 2(b), the effect of nonextensivity on the K-dV soliton structures becomes clear. Figure 2(a) and figure 2(b) show that the K-dV solitons (ϕ_1) exists with negative (positive) potential below (above) the critical value of nonextensivity (q). It has been observed that similar results were obtained by Rehman and Mishra (2016). With the increase of nonextensive parameter q, the amplitude of the positive potential K-dV solitons decreases. But the amplitude of the negative potential K-dV solitons increases as the value of nonextensivity (q) increases.

There can arise a situation in a plasma system comprising of different species such as nonthermal-nonextensive electrons, cold ion fluid, Boltzmann positrons etc., where the coefficient A vanishes at $q = q_c$ and equation (6) is unable to explain the nonlinear development of perturbation. In such a case, one needs equations involving higher order coefficients to define the system appropriately. Thus, the stretched coordinates for these higher order equations which take us to the third order calculations to derive the modified mK-dV equation and hence Gardner equation are $\xi=\varepsilon\,(x-v_p t)$ and $\tau=\varepsilon^3$ t. Now, on substituting the values of ξ and τ into equations (1)-(3) and using equation (5), we will obtain the same values of $n^{(1)}$, $u^{(1)}$and v_p as attained during the derivation of the K-dV equation (6). Further, one can utilizes $n^{(1)}$, $u^{(1)}$ and v_p and the next higher order coefficient of ε to have a set of expressions as $u^{(2)} = (\phi^{(1)})^2/2v_p^3+\phi^{(2)}/v_p$, $n^{(2)} = 3(\phi^{(1)})^2/2v_p^4+ \phi^{(2)} / v_p^2$ and $\rho^{(2)}= -A(\phi^{(1)})^2/2 = 0$ respectively.It should be noted that the above equation is satisfied identically owing to the criticality condition, i.e., $A = 0$. After replacing $\phi^{(1)}$ with φ for the sake of simplicity and rearranging the terms we get the following modified K-dV(mK-dV) equation

$$\frac{\partial\phi}{\partial\tau}+BC\phi^2\frac{\partial\phi}{\partial\xi}+B\frac{\partial^3\phi}{\partial\xi^3}=0 \tag{8}$$

Where $B=\frac{v_p^3}{2}$ and $C=\frac{15}{2v_p^6}-3\left(c_3\mu-\frac{(1-\mu)\delta^3}{6}\right)$

Here the new coefficient C and the previous coefficient B are respectively known as nonlinear and dispersion coefficients. Here, $c_3 = F_2a_1 + F_1a_2 + a_3$ and $a_3 = (q+1)(3-q)(5-3q)/48$. The steady state localized solution of equation (8) can be given as

$$\phi_2 = \phi_{2m} \sec h\left(\frac{\eta}{W_2}\right) \tag{9}$$

Where the amplitude ϕ_{2m} and width $W_2$of m-KdV solitons are given by $\phi_{2m} = \pm (6U_0/CB)^{1/2}$ and $W_1 = \phi_{2m} (C/6)^{1/2}$ respectively.

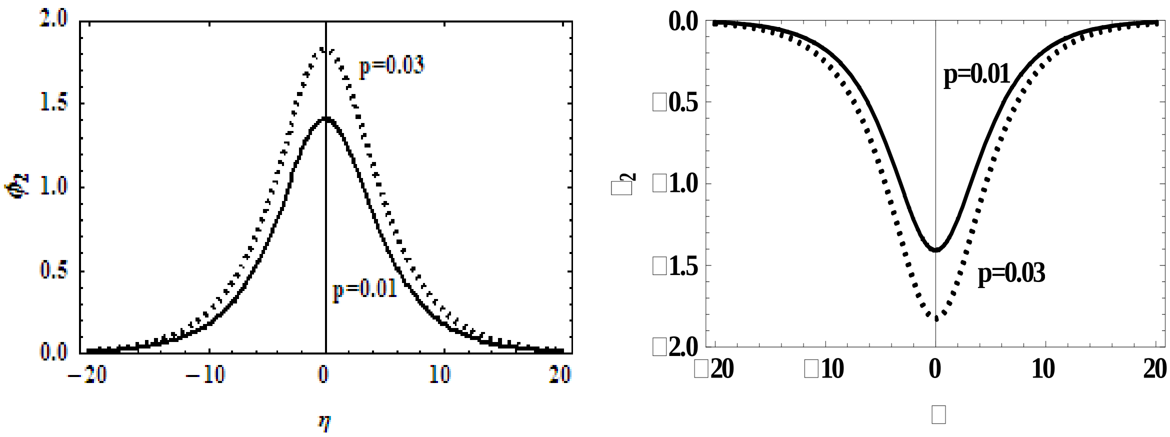

Figure 3: For Different Critical Values of q i.e. q_c= 0.85905 (p=0.01) and q_c= 0.826667 (p=0.03) with δ = 1, α = 0.1and U_0 = 0.10, plot of (a) Positive mK-dV Soliton Potential (ϕ_2) (b)Negative mK-dV Soliton Potential (ϕ_2)

From the modified K-dV equation (8), we get two values of amplitude of solitary waves. This clearly indicate that the negative and positive potential structures exist in the given plasma system at the critical values of $q = q_c$.In other words, we can say that mK-dV solitons have finite amplitudes at the critical values of nonextensivity and we get compressive or negative mK-dV solitons. In figure 3(a), for two critical values of q_c i.e. 0.85905 (p = 0.01) and 0.826667(p = 0.03) we have plotted the variation of positive potential mK-dV soliton at α = 0.1, δ = 1 and U_0 = 0.1. It is observed that the amplitude of mK-dV solitons decreases as the value of nonextensivity (q) increases. In figure 3(b), depicts the existence of negative potential mK-dV soliton for the same two critical values of q_c as shown in figure 3(a).

3. GARDNER EQUATION AND ITS SOLUTION

In order to derive the Gardner equation for the ion-acoustic waves, one can employ the second order equation of the surface charge density ρ by analyzing the ingoing solutions of equations (1)-(3) at $A = 0$ as $\phi \neq 0$, $A = 0$ gives the parametric region giving the critical nonextensive value $q = q_c$. Very close to the critical value q_c, the nonlinear coefficient $A = A_0$ can be approximated as

$$A_0 \cong s\,(\partial A/\partial q)_{q=q_c}|q-q_c| = \beta_1 s\varepsilon \tag{10}$$

Where β_1 is a constant depending on the parameter q and μ. Here,$|q-q_c|$is the small and dimensionless parameter and can be given as expansion parameter ε i.e.,$|q-q_c| \cong \varepsilon$ and s=1 for $q > q_c$ and s= -1for $q < q_c$. So, $\rho^{(2)}$can be expressed as

$$\rho^{(2)} = -A(\phi^{(1)})^2/2$$

Multiply both sides of above equation by ε^2 and substituting the value of A i.e., $A = A_0$. We get

$$\varepsilon^2 \rho^{(2)} = -\frac{1}{2}\varepsilon^3 \beta_1 s \phi^{(1)^2} = -\varepsilon^3 \frac{1}{2}\beta_1 s \phi^{(1)^2}$$

Which, therefore, must be included in the third order Poisson's equation. Here, After replacing $\phi^{(1)}$with ψ for the sake of simplicity, the next higher order in ε on simplification gives us the Gardner equation of the form:

$$\frac{\partial \psi}{\partial \tau} + \beta_1 sB\,\psi \frac{\partial \psi}{\partial \xi} + BC\,\psi^2 \frac{\partial \psi}{\partial \xi} + B\frac{\partial^3 \psi}{\partial \xi^3} = 0 \tag{11}$$

This equation contains both ψ term of K-dV and ψ^2 term of mK-dV equation. It may be noted that when $A = \beta_1 s = 0$; equation (11) reduces to mK-dV equation (8). This Gardner equation is valid for $q \sim q_c$. Moreover, the coefficients B and C are similar as in the case of K-dV and mK-dV equations. To analyse the solution of equation (11), we first consider a transformation $\eta = \xi - U_0 t$, which allows us to write Equation (11) under steady state condition, as

$$\frac{1}{2}\left(\frac{d\psi}{d\eta}\right)^2 + V(\psi) = 0 \tag{12}$$

Where the pseudo-potential

$V(\psi)$ is $V(\psi) = -U_0\psi^2/2B + \beta_1 s\,\psi^3/6 + C\psi^4/12 + \ldots\ldots$ (13)

Here, U_0 and B are always positive. Since U_0 and B are positive, so imposing the boundary conditions $[V(\psi)]_{\psi = 0}$ and $[dV(\psi)/d\psi]_{\psi = 0} = 0$ upon equation (12), we note that $[d^2V(\psi)/d\psi^2]_{\psi = 0} < 0$. These conditions imply that the solitary wave solution of Gardner equation exists if $[V(\psi)]_{\psi = \psi m} = 0$. Utilizing this condition in equation (13), we have

$0 = -U_0/2B + \beta_1 s\,\psi_{m1,2}/6 + C\psi^2_{m1,2}/12 + \ldots\ldots$ (14)

So, the two roots of the quadratic equation (14) can be given as $\psi_{m1,2} = \psi_m[1 \pm (1+ U_0/V_0)^{1/2}]$ where $\psi_m = -\beta_1 s/C$ and $V_0 = \beta_1^2 s^2 B/6C$. Now using the value of $\psi_{m1,2}$ and equation (13) in equation (12), we get

$$\left(\frac{d\psi}{d\xi}\right)^2 + 2\psi^2(\psi - \psi_{m1})(\psi - \psi_{m2}) = 0 \tag{15}$$

Thus, the soliton solution of Gardner equation can be given as

$$\psi = \left[\frac{1}{\psi_{m2}} - \left(\frac{1}{\psi_{m2}} - \frac{1}{\psi_{m1}}\right)\cosh^2\{\frac{\eta}{\mathrm{W}}\}\right]^{-1} \tag{16}$$

Where $W = (2/-\psi_{m1}\psi_{m2})^{1/2}$ is the width of the Gardner solitary waves. Equation (16) provides a solution of the Gardner solitons or Gardner solitary waves which have been represented graphically in figures 4(a) and 4(b) respectively.

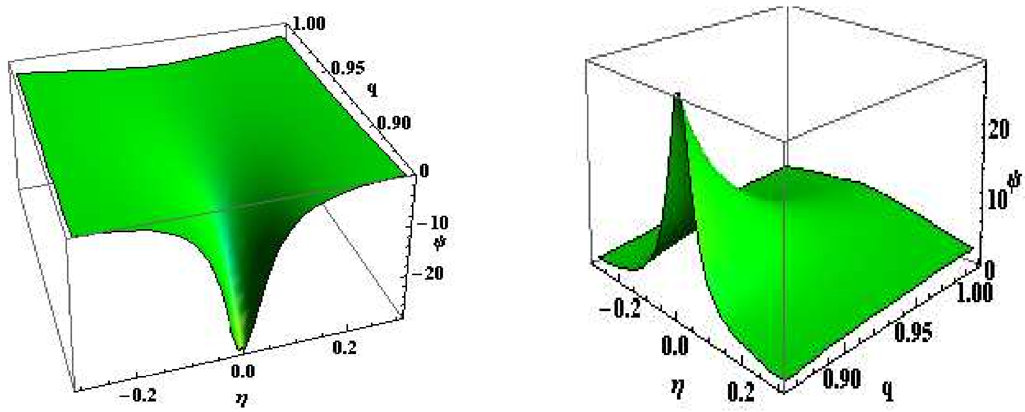

Figure 4: For Two Different Critical Values of q for p = 0.01 and, δ = 1, α = 0.1and U_0 = 0.10. (a) Plot of Rarefactive Gardner Soliton Potential (ψ). (b) Plot of Compressive Gardner Soliton Potential (ψ).

The basic features of the solitary profiles represented by the Gardner solitary waves of Gardner equation are very useful to explain the characteristic features of the nonlinear phenomena.Close to the critical value of q_c, if K-dV equation is used to study soliton solution, for a parametric regime equivalent to the coefficient of nonlinear term of K-dV equation i.e., A ~ 0, then it corresponds to infinitely large amplitude soliton. Thus, the validity ofreductive perturbation methodbreaks down. In such case, we consider another types of nonlinear dynamical equation termed as Gardner equation, that can be valid for $q \sim q_c$. In figure 4(a), the variation of negative potential Gardner solitons with q for p = 0.01, δ = 1, S = -1 and U_0 = 0.1 have been plotted. It is clear from figure 4(a) that the negative potential (dip shape) solitary waves exist just below the critical value of q. In figure 4(b), the variation of positive potential Gardner solitons with q for p = 0.01, δ = 1, S = 1 and U_0 = 0.1 have been plotted. When S = 1, i.e., $q < q_c$ Equation (16) represents a bright solitons, whereas when S = -1, i.e., $q > q_{c,}$ (16) represents a dark solitons. Similar results were obtained by Deeba et al. (2012). Also, we note from figure 4(b) that the positive potential (hump shape) solitary waves exist just above the critical value of q. The amplitude of Gardner solitons increases as the nonextensive parameter q increases.

4. CONCLUSIONS

In the nonextensive and non-thermal environment of electrons, a study of electron-positron-ion (e-p-i) plasma system is presented here. Using the reductive perturbation method, nonlinear differential equations viz, K-dV; modified K-dV and Gardner equations have been derived. The wave phase velocity, nonlinearity and dispersion coefficient are found to be the functions of various parameters such as nonextensivity (q), free electron to positron temperature ratio (δ), nonthermality (α) and positron to electron number density (p) respectively. The graphical representations of various numerical results have been presented in the various parameter regimes. It is found that phase velocity decreases with nonextensivity and increases with nonthermality (α). Further, the critical value of nonextensivity changes with nonthermality and increases as α increases Peak amplitude of positive potential K-dV structures increases whereas amplitude of negative potential K-dV structures decreases with increase in nonextensivity. At the critical value of nonextensivity i.e., q_c, K-dV soliton cease to exist. However, positive (negative) mK-dV potential structure exist at q_c. Moreover, bright (dark) Gardner solitons exist for $q < q_c$ ($q > q_c$). This model may be helpful to understand the nonlinear features of electrostatic perturbations observed in numerous interstellar space plasmas such as pulsar environments, auroral acceleration regions, active galatic nuclei, supernovas as well as in laboratory plasmas in which Boltzmann positrons and nonthermal-nonextensive electrons are the major plasma species. Results of this investigation are similar to those obtained by Deeba et al. (2012) and Rehman and Mishra (2016).

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